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**COACHING CENTRE FOR PHYSICS**

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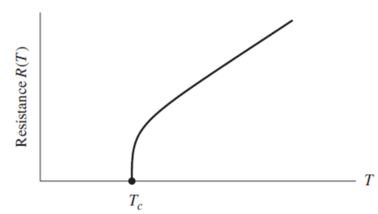
## SUPERCONDUCTIVITY

All metals are good conductors since they are having free electrons. The resistance of a metal is due to the scattering of conduction electron by lattice vibrations hence when temperature increases (i.e vibration increases) its resistance also increases

Some metals and alloys cooled to sufficiently low temperatures can conduct electric current without resistance. So these materials undergo a phase transition to a new superconducting state having NO D.C. resistance (**thus at $T_C$ zero resistivity i.e. infinite conductivity is observed.)**

In pure metals the zero resistance state can be reached within a temperature range of 1 mK. In the case of impure metals the transition to the superconducting state may be considerably broadened.

The phenomenon of superconductivity was first discovered by, Kamerling ones in 1911 when he was measuring the resistivity of a pure Mercury at low temperature he observed that the resistivity of Mercury drops suddenly to zero at about **4.2 Kelvin** shown in figure




He concluded that Mercury possess a new state is called a superconducting state.

The temperature at which resistance disappears is called transition temperature or critical temperature.

- Below $T_C$ materials behaves as superconductor
- About $T_C$ materials behaves as normal conductor
- $T_C$ is different for different substances
- For pure and structural solids at $T_C$ Sharp.
- For impure and imperfect structural solids we have a range of transition temperature (i.e $T_C$ is broad.)

The super conductivity is observed in metals and alloys but not in Ferro and antiferromagnetic materials.

## DIFFERENT EFFECTS ON SUPERCONDUCTOR

### ISOTOPIC EFFECT (Ionic Effect)

It has been observed that the critical temperature of superconductors varies with isotopic mass.

The relation between isotopic mass and transition temperature is given by

$$M^{\alpha} T_c = const$$

Here $\alpha$ is a number usually 0.5

$$M^{1/2} T_c = const$$

$$T_c \; \alpha \; \frac{1}{M^{\frac{1}{2}}} \qquad T_c \downarrow \text{ as mass} \uparrow$$

Heavier isotopic mass having lowest lattice vibration therefore the transition temperature $T_C$ decreases with increasing isotopic mass. This implies that the superconductivity is due to the interaction between **Electron - Lattice - Electron** interaction

Different isotope having different transition temperature

### EFFECT OF MAGNETIC FIELD

The phenomenon of superconductivity can be destroyed by applying sufficiently large magnetic field.

The minimum field which required to regain the normal conductivity is called a "critical field" .

The magnetic field which required to destroy the phenomenon of super connectivity is called the critical field.

- Below $T_C$ super conductivity exist
- Above normal conductivity exist

The variation of critical magnetic field with the temperature is shown in figure

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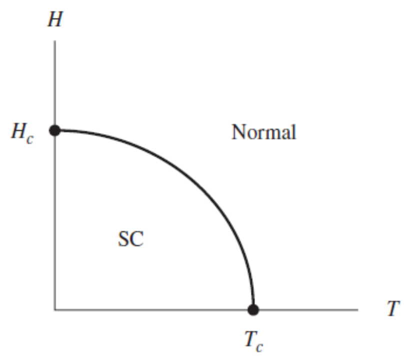
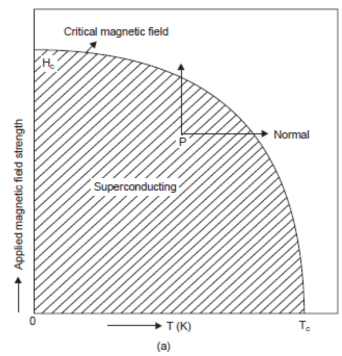



From the fig it is clear that the critical field required to destroy superconducting state is progressively decreased with increase in temperature.

Mathematical relation between $H_C$ and $T_C$ is

$$H_C(T) = H_C(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

$H_C(0)$ a critical field at $T = 0K$

**Effect of electric current**

The critical magnetic field required to destroy superconductivity need not necessarily be applied externally.

An electric current flowing a superconducting wire, it gives rise to its own magnetic field. As the current is increased to a critical value $i_c$., the associated magnetic field becomes $H_c$ and the superconductivity disappears.

The minimum current which can destroy the superconducting state is called critical current.

Let consider a superconducting wire or radius $r$ *and* $i$ be the current through the wire.

The critical current ($i_c$) required to destroy the superconducting property is given by

$$i_c = 2\pi r H_c.$$

Here, $H_c \rightarrow$ critical magnetic field.

$r \rightarrow$ radius of the superconducting wire

The maximum possible current that flows through superconductor is limited by above equation . Hence this is the main barrier for NOT producing high field superconducting magnets.

$J < J_C$ The material in the superconducting state.

From the above *phase diagram* of a superconductor. The metal will be a superconducting one for any combination of the applied magnetic field and temperature. This combination gives a point P. The arrows indicate, the metal can be driven into the normal state by increasing either the field or temperature.

If $H_C(T)$ is the strength of the magnetic field required to destroy the superconductivity in a specimen at a temperature $T$ .
The magnetic induction B and the external magnetic field $H_0$ are related by

$$B = H_0 + 4\pi M$$

$$\text{As } H_0 < H_c \text{ then } B = 0$$

$$M = -\frac{H_0}{4\pi}$$

Work must be done to add $dH_0$ to the magnetic field $H_0$. This work is stored as the free energy of the superconductor

$$-M\, dH_0 = \frac{H_0\, dH_0}{4\pi}$$

When the field is changes from $0$ to $H_0$ the work done by the source is

$$-\int_0^{H_0} M\, dH_0 = \int_0^{H_0} \frac{H_0\, dH_0}{4\pi}$$



$$-\int_0^{H_0} M\, dH_0 = \frac{{H_0}^2}{8\pi} + F_{s0}$$

Where $f_{s0}$ is the free energy density in superconductor in a zero magnetic field.

The above work is stored as free energy of the superconductor

$$F_{sH} - F_{s0} = \frac{{H_0}^2}{8\pi}$$

At $H_0 = H_c$ (transition from super conducting state to normal state) $F_{sH} = F_n$

$$F_n - F_{s0} = \frac{{H_c}^2}{8\pi}$$

The difference in the *free energy* per unit volume between the superconducting state and the normal state at this temperature is given by $\frac{{H_c}^2}{8\pi}$ (OR)

$$F_n - F_{s0} = \frac{{H_c}^2}{8\pi}$$

This energy difference is called the *condensation energy*

**Persistent current**

When the superconductor in the form of a ring is placed in a magnetic field, then a current is induced in it by electromagnetic induction.

In case of normal conducting ring, the current decreases quickly because of the resistance of the ring.

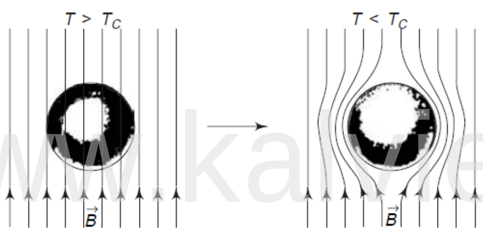
If the ring is in superconducting state, the ring has zero resistance. Once the current is set up, if flow indefinitely without any decrease in its value.

This current persists even after the removal of the magnetic field is called persistent current.

This current exists without any decrease in its strength, as long as the material is in superconducting state.

**MEISSNER EFFECT**

The expulsion of magnetic lines of force from superconductor is called Meissner effect i.e If $T < T_C$ the flux is totally pushed out from the superconductor




At $T = T_C$ the flux penetrate suddenly in the material in then the material converted into superconductor to normal conductor.

The magnetic induction inside the solid

$$B = \mu H$$

$$B = \mu_0 \mu_r H$$

$$B = \mu_0 (1 + \chi) H$$

$$T < T_C, \quad B = 0$$

$$\mu_0 (1 + \chi) H = 0$$

$1 + \chi = 0,\ \chi = -1$ (OR) $\chi = \frac{-1}{4\pi}$

i.e Perfect diamagnetism is an essential property of superconducting state.

$$B = \mu H = \mu_0 \mu_r H$$

$$B = \mu_0 \mu_r H = 0$$

$$\therefore\ \mu_r = 0$$

And $B = 0$. Hence

$$\mu_0(H + M) = 0$$

Since $\mu_0 \neq 0, \quad H + M = 0$

Or $M = -H$

Magnetic susceptibility, $\chi = \frac{M}{H} = -1$

Thus the material is perfectly diamagnetic.

We know a perfect conductor cannot sustain an electric inside that is $E = 0$, it therefore from Maxwell Equations (Faraday's equation)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

If $E = 0$ then $\frac{\partial B}{\partial t} = 0$

The magnetic flux density $B$ inside a superconductor should be constant. This means that if a perfect conductor is placed in a magnetic field and then cooled down to $T_C$ the magnetic flux remains trapped inside even when the field is removed.

Meissner and Ochsenfeld however observed that it does not happen in a superconductor. They found that the flux is expelled from the body of the superconductor the moment it is cooled down to below $T_C$. Irrespective of the fact whether the superconductor is kept in a magnetic field and cooled below $T_C$ or it is cooled below $T_C$ first and then a field is applied, the magnetic flux does not enter a superconductor.

Thus a superconductor does not obey Maxwell equation.

**Thermal Properties of Superconductors**

**ENTROPY**

The *free energy density* $F_{sH}$ of the metal in the superconducting state is given by $\frac{{H_C}^2}{8\pi}$

From the first law of thermodynamics

$$dQ = dU + dW$$

From the definition of free energy

$$F = U - TS$$

$$dF = dU - TdS - SdT$$

From second law of thermodynamics

$dQ = TdS$

$$dF = dU - dQ - SdT$$

$$dF = -dW - SdT$$

So $S = -\left(\frac{\partial F}{\partial T}\right)_W$

$$S = -\left(\frac{\partial F}{\partial T}\right)_W = -\frac{H_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)_W$$

OR

$$S_n - S_s = -\mu_0 H_c\left(\frac{\partial H_c}{\partial T}\right)_W$$

The difference in entropy between normal and superconducting state can be written as $S_n - S_s = -\mu_0 H_c\left(\frac{\partial H_c}{\partial T}\right)_W$

$$\text{If } \frac{dH_C}{dT} = -ve$$

Because from the temperature $(T)$ versus magnetic field $(H_0)$ graph. $H_C$ monotonically descending as temperature increases.

$$S_N - S_S = +ve \qquad [S_N > S_S]$$

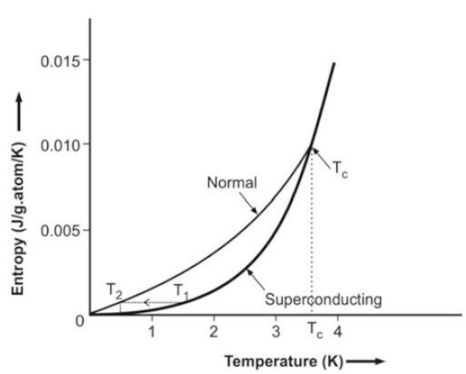
Superconducting state is more order than the normal conducting state.

Entropy of all superconductors decreases considerably upon cooling below $T_C$.

The entropy is a measure of the degree of disorder of a given system and hence this decrease in a superconductor signifies that the superconducting state is more ordered than the normal state.

The fraction of electrons that is thermally excited in the normal state becomes ordered in the superconducting state

The variation of Entropy for normal and superconducting state as a function of temperature is shown in figure




$$T = T_C, \qquad H_C = 0$$

$$S_N - S_S = 0$$

$$S_N = S_S$$

i.e according to third law of thermodynamics at $T = 0k$ irrespective of any material the entropy is zero.

The entropy variation is of the order of $10^{-4}\ k_B/atom$

**SPECIFIC HEAT**

The specific heat in normal conductor consists of two parts

$$C_V = C_V^e + C_V^l$$

$$C_V = \gamma T + \beta T^3 \quad \text{for normal state}$$

The first term linearly proportional to *T* while the second term is proportional to $T^3$.

Electronic part is linearly proportional to *T*
And Lattice part is proportional to $T^3$

In super conducting state $C_V = \beta T^3 + \gamma\, e^{-E_g/2K_BT}$

The expression for entropy is $S = -\left(\frac{\partial F}{\partial T}\right)$

At $T = T_C$ , $S_S = S_n$

$$\left(\frac{\partial F}{\partial T}\right)_s = \left(\frac{\partial F}{\partial T}\right)_n$$

*A phase transition* which satisfies this condition is known as *second order phase transition.*

Characteristics of second order transition:

1. At the transition there is no latent heat,
2. There is a jump in the specific heat.

The specific heat of a material is given by

$$C = v\,T\left(\frac{\partial S}{\partial T}\right)$$

Where $v$ –the volume per unit mass, so the difference in the specific heats of the superconducting and
normal states is

$$C_s - C_n = v\,T\mu_0 H_c\left(\frac{d^2H_C}{dT^2}\right) + v\,T\mu_0\left(\frac{dH_C}{dT}\right)^2 \qquad \because S_s - S_n = \mu_0 H_c\left(\frac{\partial H_c}{\partial T}\right)$$

At the transition temperature$T = T_C$, $H_C = 0$ and so

$$C_s - C_n = v\,T_c\ \mu_0\left(\frac{dH_C}{dT}\right)^2_{T_c}$$

This is the famous *Rutger's formula*, and it predicts the value of the discontinuity in the specific heat of a superconductor at the transition temperature.

If $v = 1$, then

$$C_s - C_n = T_c\ \mu_0 \left(\frac{dH_C}{dT}\right)^2_{T_c}$$

The specific heat difference of normal and superconductor at $T = T_C$ is given by

$$C_s - C_n = T_c\ \mu_0 \left(\frac{dH_C}{dT}\right)^2_{T_c}$$

$$\frac{dH_C}{dT} = \text{is always} - ve$$

But $$C_s - C_n = +ve$$

$$C_S > C_N$$

***Specific heat show certain Jump at $T_C$.***

The specific heat $C$ in a normal metal consists of two contributions, $C_e$ from the electrons in the conduction band and $C_l$ from the lattice.

For metal, $(C)_N = (C_l)_N + (C_e^-)_N$ $N$ –normal state

$(C)_s = (C_l)_S + (C_e^-)_S$ $S -$ super conducting state

Structural property does not change in super conducting state

$$(C_l)_S = (C_l^-)_S$$

$$C_N - C_S = (C_e^-)_N - (C_e^-)_S$$

Where, The specific heat $C_N$ in a normal state

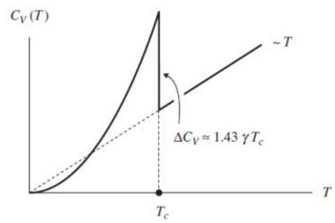
The specific heat $C_S$ in super conducting state

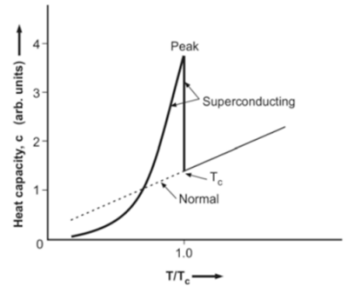
At $T < T_C$, $H_C \neq 0$ and so

$$C_s - C_n = -ve$$
$$C_S < C_N \ \ or\ \ C_N > C_S$$

i.e specific heat decrease in super conducting state compare to normal state. Because as temperature decreases specific heat decreases.

The variation of specific heat of superconductor and normal conductor is shown in fig




**Thermal Conductivity**

The magnitude of thermal conductivity from normal state to the superconducting state at large amount.

In a normal metal heat flow is totally by the conduction electrons.

The *super electrons* in the superconducting state not interact with the lattice so they can not *exchange energy*, i.e they cannot pick up heat from one part of the specimen and deliver it to another. Or, if a metal goes into the superconducting state, its *thermal conductivity* is reduced.

*Thermal conductivity* is reduced well below the critical temperature as very few normal electrons are available to transport thermal energy.

By the application of a magnetic field, the thermal conductivity is restored to the normal (state) value. Hence the thermal conductivity of superconductor can be controlled by means of a magnetic field, and this effect has been used in "*thermal switches*" at low temperatures to make and break heat contact between specimens connected by a link of superconducting materials.

$EX$:The thermal conductivity of tin at 2 $K$ is 34 $W\ cm^{-1}\ K^{-1}$ for the normal phase and At 4 K(superconducting phase), it is 55 $W\ cm^{-1}\ K^{-1}$ ($T_C$ of tin 3.73 $K$).

**Thermoelectric Effects**

The thermoelectric effects do not occur in a superconducting metal. i.e no current is set up around a circuit consisting of two superconductors, if the two junctions are held at different temperatures below their transition temperatures.

Hence there is *no* thermo $e.m.f$ in superconducting circuits, the Peltier and Thomson coefficients must be the same for all superconducting metals, and they are in fact zero.

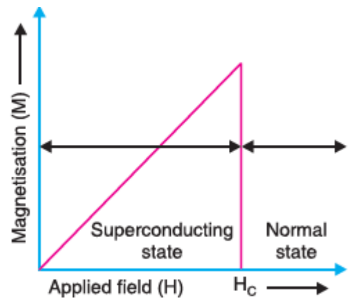
**<u>TYPE 1 AND TYPE 2 SUPERCONDUCTORS</u>**

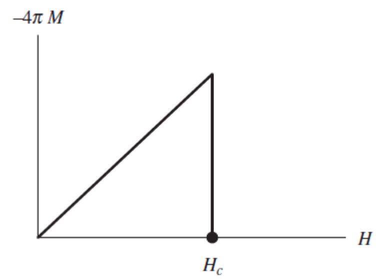
The superconductors can be classified on the basis of magnetic behaviour namely Type 1 and type 2

**TYPE 1** : The material in which the transition from superconducting state to normal conducting state occurs suddenly at critical field $H_C$ is known as type1 superconductor.

i.e *The superconductors, in which the magnetic field is totally excluded from the interior of superconductors below a certain magnetising field $H_C$, and at $H_C$ the material loses superconductivity and the magnetic field penetrates fully into the material are called type I or soft superconductors.*

The dependency of magnetisation and magnetic field is shown in figure




$$Ex: Hg, Ni, Sn, Al, Zn, Mg, Pb \dots\dots$$

**Characteristics:**

1. Type I Superconductors are perfectly diamagnetic below $H_C$
2. They exhibit complete Meissner effect.
3. The maximum critical field for type I superconductor is of the order of 0.1 T. Due to low $H_C$ these materials unsuitable to make high field superconducting magnets.
4. The magnetisation curve shows that transition at $H_C$ is reversible. This means that if the magnetic field is reduced below $H_C$, the material again acquires superconducting property and the field is expelled.
5. Below $H_C$ the material is superconductor and above $H_C$ it becomes a conductor.
6. Type 1 superconductors are also called soft superconductors

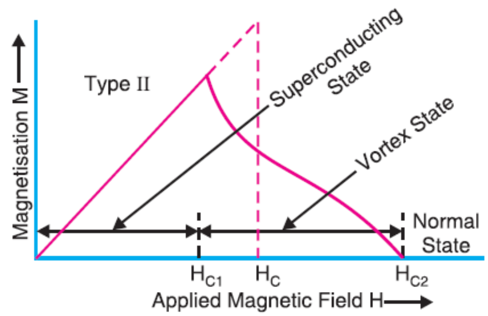
**TYPE 2 superconductors (Hard) :**

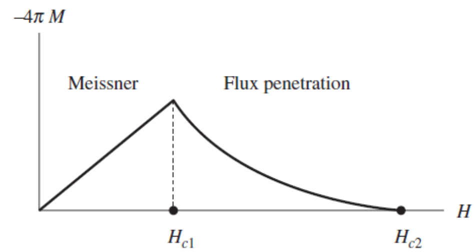
The superconductors which are characterized by two critical field is called type 2 superconductor $H_{C1}, H_{C2}$

The transition superconducting state to normal conducting occurs gradually as magnetic field increased from $H_{C1}$ to $H_{C2}$ .

i.e *The superconductors in which the material loses magnetisation gradually rather than suddenly are termed as type II or hard superconductors*.

The dependency of magnetisation and magnetic field is shown in figure




**Characteristics:**

1. They do not show complete Meissner effect.
2. These superconductors have two critical fields: $H_{C1}$ (the lower critical field) and $H_{C2}$ (the upper critical field). The specimen is diamagnetic below $H_{C1}$ *i.e.*, the magnetic field is completely excluded below $H_{C1}$.
   (Below $H_{C1}$ type II Superconductor are perfectly diamagnetic)
3. At $H_{C1}$ the flux penetrate partially through the superconductor this penetration of flux lines is gradually increased as a field increases from $H_{C1}$ to $H_{C2}$.
4. At $H_{C2}$ to the material totally changes from superconducting state to normal conducting state.
5. The region between $H_{C1}$ to $H_{C2}$ is intermediate (or) mixed state (or) vortex state. In mixed state the material is in magnetically mixed state but electrically in superconducting state.

Type 2 superconductors are technically advantage than type 1 Superconductors

The type 1 of superconductors or soft superconductors and type 2 superconductors are hard superconductors.

The value of critical field for type II materials may be 100 times or more higher than the value of $H_c$ for type I superconductors.( $H_{C2}\ upto\ 30T$)

Ex: Nb–Zr, Nb–Ti alloys and Va-Ga and Nb-Sn inter-metallic compounds.

Lower critical magnetic field

$H_{c1} = \frac{\emptyset_0}{\pi\lambda^2}$ $\lambda < \varepsilon_0$

Higher critical magnetic field

$H_{c2} = \frac{\emptyset_0}{\pi{\varepsilon_0}^2}$ $\lambda > \varepsilon_0$

Where $\emptyset_0$ –fluxoid

**LONDON'S EQ UATION**

Electrical conduction in the normal state of a metal is described by Ohm`s law $j = \sigma E$.

let $n_s$ be the number of super electrons at temperature less than $T_C$

The super current density is

$I = nAeV_d \qquad J = I/A$

For $e^-$ $\qquad J = nev$

$J_S = -n_s eV_s$ ------------------ (1)

The motion of super electron in the electric field is given by

$$ma = -eE$$

$$m.\frac{dv_s}{dt} = -eE - - - - - - -(2)$$

Differentiate equation (1) $\qquad \frac{dJ_S}{dt} = -n_s e\frac{dv_s}{dt}$

$$= -n_s e\left(-\frac{eE}{m}\right)$$

$$\frac{dJ_s}{dt} = n_s\left(\frac{e^2E}{m}\right) \qquad \dots\dots\dots\dots\dots \; I^{st} Londen\ equ$$

$$If\ E = 0,$$

$\frac{dJ_S}{dt} = 0 \qquad \therefore J_S \rightarrow finite\ constant\ value$

It is possible to have the steady state current in a superconductor even in the absence of external electric field this explain the phenomenon of superconductivity.

For normal conductors

$$J = \sigma E$$

$$J = 0 \quad [\therefore E = 0]$$

**II[nd] LONDON'S EQUATION**

All Maxwell equ $\qquad \nabla \times E = -\frac{\partial B}{\partial t}$

$$If\ E = 0,$$

$$\frac{\partial B}{\partial t} = 0, \quad B = constant$$

i.e B is constant inside a superconductor irrespective of temperature .This contradicts with meissner effect.

According to which is superconductor expel out the magnetic flux completely for all temperature below $T_C$.

In order to remove this difficulty London corrected above formula by taking curl of first equation

$$\nabla \times \frac{dJ_s}{dt} = -\frac{n_s e^2}{m} \nabla \times E$$

$$\nabla \times \frac{dJ_s}{dt} = -\frac{n_s e^2}{m}\left(-\frac{\partial B}{\partial t}\right)$$

$$\nabla \times J_S = -\frac{n_s e^2}{m} B \qquad \ldots\ldots\ldots\ldots\ldots\ldots . II\ london\ equ$$

$$\nabla \times J_S = 0$$

$J_S$ is irrotational

This explains meissner effect. The above equation is second London equation

**PENETRATION DEPTH**

Ampere`s law $\nabla \times B = \mu_0 J_S$

$$\nabla \times (\nabla \times B) = [\nabla\,(\nabla . B) - \nabla^2 B] = \nabla \times (\mu_0 J_S)$$

$$-\nabla^2 B = \mu_0 \nabla \times (J_S)$$

$$-\nabla^2 B = -\mu_0 \frac{n_s e^2}{m} B$$

$$\nabla^2 B = \frac{\mu_0 n_s e^2}{m} B$$

$$\nabla^2 B = \frac{1}{\lambda^2} B$$

This is a II order different equation hence solution is $B(x) = B(0)e^{-x/\lambda}$

$$\lambda^2 = \frac{m}{\mu_0 n_s e^2}$$

$$\lambda_L\ (or)\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

Where $\lambda_L$ is a constant with the dimensions of length and it is called the London penetration depth.

$$B(x) = B(0)e^{-x/\lambda}$$

From this flux density decreases experimentally inside a superconductor

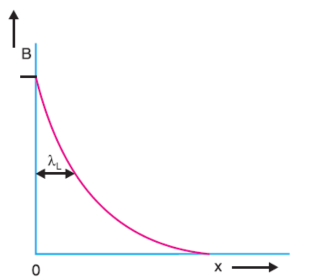
$$B(x) = B(0)e^{-x/\lambda}$$

$B(x) = B(0)\left(\frac{1}{e}\right) \qquad \therefore x = \lambda$

This is the magnetic flux density at the surface of a conductor.

From this flux density decreases exponentially inside a superconductor and falling to $\frac{1}{e}$ value at a distance of $\lambda$.

The dependence of $\lambda$ according to temperature T is given by




The penetration depth $\lambda_L$ is defined as the distance in which the field decreases by the factor $e^{-1}$.

$\lambda$ increases with increase of temperature and become infinite at $T = T_C$

i.e The substance becomes normal and the field penetrates the whole specimen.

The relation between penetration depth and no of super electron is inversely proportional and also temperature-dependent.

The number of super electron is given by

$n_S = n_0\left[1 - \frac{T^4}{{T_C}^4}\right]$

At 0K, $\quad n_s = n_0 \qquad$ At $T = T_C \quad n_S = 0$ (All the $e^-$ are normal)

So the penetration depth is not constant but varies widely with temperature.

$$\lambda(T) = \frac{\lambda_0}{\left[1 - \left(\frac{T}{T_C}\right)^4\right]^{\frac{1}{2}}}$$

$\lambda_0 = 30 - 130\, nm$

The variation of super electron, penetration depth and normal electrons with $T_C$ is shown in figure

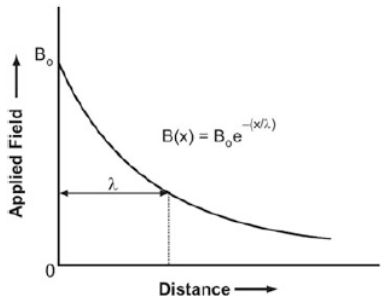



Fig: Field penetration in a superconductor. The magnetic flux drops exponentially inside the material. Penetration depth λ is defined as the depth at which the flux density drops to its $\frac{1}{e}th$ value.

**Coherence length:**

In the *condensed state(super conducting state)*, the velocities of a pair of electron are correlated only if the distance between them is less than a certain coherence length $\varepsilon_L$. *This is a measure of the size of the cooperpair*.

The electrons which participate in the condensation process, have an energy within a range Δ (or) $E_g$ of the Fermi energy.

The resulting momentum $\Delta P = \frac{2\Delta}{V_F}$ (OR) $\Delta P = \frac{2E_g}{V_F}$

with $V_F$ the velocity at the Fermi level. By *uncertainty principle*

$$\varepsilon_L \Delta P = \hbar$$

$$\varepsilon_L = \frac{\hbar}{\Delta P}$$

$$\varepsilon_L = \frac{\hbar V_F}{2\Delta} \quad (OR)\ \varepsilon_L = \frac{\hbar V_f}{2E_g}$$

Paired $e^-$ is not scattered, as a result they travel at sudden distance in coupled manner is coherence length $10^{-4} cm\ (or)\ 10^{-6}\ m$

$V_F$ = Velocity of Fermi electron

The ratio of penetration depth to Coherence length is $\frac{\lambda}{\varepsilon_L} = k$ (where $k$ is a dimensionless constant)

For Type I s.c. $k > \frac{1}{\sqrt{2}}$ For Type II s.c $k < \frac{1}{\sqrt{2}}$

**BCS theory**

The microscopic theory of superconductivity was put forward by Barden, Cooper, Schrieffer in 1957.

This theory provides a quantum of superconductivity and explains all the properties exhibited by superconductors.

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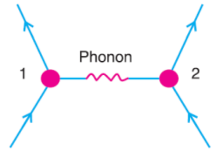
BCS showed that the basic interaction which is responsible for superconductivity is the pairing of electrons by means of exchange of virtual phonons.

**Electron Lattice Electron Interaction**




$e^- + (attraction)\ another\ e^-$ via lattice

To understand this consider an electron approaches a $+ve$ ion core then it suffers columbic interaction due to this attraction, ion core has certain motion and consequently the lattice gets distorted.

At the same time another $e^-$ approaches this distorted lattice then the interaction between the electrons and the lattice occurs.

**<u>EXPLANATION</u>**

If wave vector of incident $e^-$ is $\overrightarrow{K_1}$ of wave. When this $e^-$ scattered by the lattice it transfer a virtual phonon to the lattice.

That is the wave vector of the scattered $e^-$ is $(\overrightarrow{K_1} - q)$ where $q$ is the wave vector of the virtual phonon transferred to the lattice.

After getting a virtual phonon, lattice goes in to excited state.

When an another $e^-$ of wave vector $\overrightarrow{K_2}$ scattered by the excited lattice then this virtual phonon is transferred to this $(2^{nd}\ e^-)$ and now it scatters with a wave vector $(\overrightarrow{K_2} + q)$.

Hence there is a net transfer of energy and momentum from $1^{st}\ to\ 2^{nd}\ e^-$ Via deformed lattice.

If this exchange energy is more than columbic repulsion $b/w$ two electron then two $e^-$ can form a bounded pair and this bound pair is known as cooper pair.

From this we understand that two electrons are interact via distorted lattice. This type of interaction is called the Electron- Lattice- Electron interaction.

This interaction is strong when the two electrons have equal and opposite momentum and spin. (i.e., $k \uparrow -k \downarrow$)

Total spin is zero (i.e Angular momentum also zero)

The attraction of two electrons via phone on field (virtual phonon) is called **"cooper pair"**. These electrons are paired to form a single system called Cooper pair. Their motions are correlated.

As the two electrons attract and form a bound pair they behave like a Boson obeying Bose- Einstein statistics. All the Bosons can then condense coherently into a single quantum state.
(But actually the electrons are fermions and obey Fermi-Dirac statistics according to which no two electrons can occupy the same quantum energy state)

The Confirmation of the electron-phonon (lattice) interaction responsible for superconductivity came from the discovery of isotope effect

$$M^{1/2}T_c = const$$

Thus it become clear that the lattice of ions in a metal actively participating in the creation of super conducting state.

They(Cooper pair of electrons) tend to come together via the exchange of a virtual phonon and form a bound state. These pairs, called Cooper pairs have a binding energy of $10^{-3}\ TO\ 10^{-4}\ eV$.

To preserve this binding energy against thermal excitations it is necessary to maintain the temperature low.

The binding is strongest when the electrons have equal and opposite momentum so that the total momentum of the pair is zero and the pairs are in spin singlet state.

The size of the Cooper pair, *'r'* is

$$r = \frac{\hbar v_f}{E_g}$$

Where $E_g$ is the binding energy and $v_f$ the Fermi velocity.
By taking the $E_B$ as $10^{-3}\ TO\ 10^{-4}\ eV$ and $v_f = 10^8\ cm/s$ in a metal,

We get $r = 100\ to\ 1{,}000\ nm$

Which is the same as lattice distortion *'d'*

$$d \approx v_f \frac{2\pi}{\omega_D}\ 100\ to\ 1{,}000\ nm$$

The energy of the pair of electron in the bounded state is less than the energy of the pair in the free state. i.e they are able to form a bound state so that their total energy is less than $2E_F$.

The maximum Wave length of radiation required to destroy superconductivity

$$\lambda = \frac{hc}{E_g} = \frac{12400}{E_g\ (eV)} A^\circ$$

***At low temperature less than the $T_c$ Electron Lattice Electron interaction is stronger than electron electron coulomb interaction.***

$$(e^- - L - e^-) > (e^- - e^-)$$

As a temperature approaches to $T_c$ $(i.e\ T \to T_c)$ the valance $e^-$ tends to pairup and paring is complete at $T_c = 0k$

The paring is completed broken at $T = T_c$

The Cooper pair may be considered as a new particle with mass and charge twice that of electron. This can be represented by a single particle (hence single debroglie wave.)

i.e copper pair charge $= 2e$ & mass $= 2m_e$

The momentum of cooper pair is small so that the wavelength becomes larger so that the probability of scattering is small it leads to infinite conductivity. So it move in the lattice without resistance.

**Coherence length** is defined as the distance over which two electrons combine to form a Cooper pair. In other words, it is the smallest dimension over which superconductivity can be established or destroyed.

**Flux quantization in a superconducting ring**

A magnetic flux inside a superconducting cylinder or in a ring will always be an integral multiple of $(h/2e)$ where h is the plank constant and e the electronic charge.

Magnetic flux $\phi$ through a superconducting ring is quantized as i.e.,

$$\phi = n\phi_0$$

$$\phi = n\frac{h}{2e}$$

Here, $n$ is an integer and

$\phi_0 = \frac{h}{2e} = 2.068 \times 10^{-15}\ Tm^2$ is referred to as a quantum fluxoid.

The flux quantization involves in terms of charge $2e$ which is due to the consequence of BCS theory.

**ENERGY GAP**

The exponential increase of specific heat at $T = T_C$ implies the presence of an energy gap in the super conducting state. This energy gap is entirely different than energy gap of an insulator.

$$({C_e}^-)_S = Ae^{-\Delta/uT}$$

$\Delta\rightarrow$ Energy gap which separates normal & superconducting state

According to BCS theory relates the energy gap of a superconductor at $0\ K$ at critical temperature $T_c$ is

$$\Delta\ (or)\ E_g(0) = 2b\ k_B T_C$$

$\Delta\ (or)\ E_g(0) = 3.5\ k_B T_C$ $\qquad 2b = 3.5$

- The Energy gap at ok $3.5k_B T_C$
- The Energy gap at $T_C\rightarrow$ is zero

There is a gap in superconductor that separate superconducting electrons from normal electron state.

$\Delta$ is separation between $e^-$& super $e^-$

At $T_C = \Delta$ is zero

The energy gap in superconductor is continuously decreases to zero as a temperature approaches zero to $T_C$ but in insulator the energy gap separates valence band and conduction band is nearly independent of temperature.

At temperatures above $0K$, some Cooper pairs break up. At the critical temperature $T_c$, the energy gap disappears, there are no more Cooper pairs, and the material is no longer be superconducting.

$E_g$ *is typically* $= 10^{-4} eV$

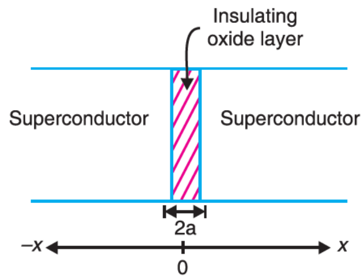
Cooper pair can be broken into quasi-electrons only when an energy equal to $2\Delta$ is supplied to the superconductor. These electrons will occupy the empty levels above the energy gap.

At finite temperature (below $T_C$) bound pairs as well as single electrons are distributed. All the pairs occupy the ground state and the single electrons occupy the levels above the energy gap.

BCS theory that accounts for all the experimental observations namely,
(i) the zero electrical resistance below $T_C$,
(ii) second order phase transition at $T_C$,
(iii) Meissner effect,
(iv) an energy gap of right magnitude,
(v) a jump in specific heat at $T_C$ and an exponential term in the electronic specific heat in superconducting state,
(vi) dependence of $T_C$ on isotopic mass and
(vii) the penetration depth and its temperature variation.

**JOSEPHSON EFFECT (or) JOSEPHSON JUNCTION**

Josephson experimentally predicted that a *'super current' consisting of correlated pairs of electrons can be made to flow across an insulating gap between two superconductors provided the gap is small enough.*




Consider the sandwich arrangement of two superconductor separated by a thin insulator layer of 10 to 20 $A^{\circ}$ thickness this is known as Josephson Junction.

In 1962 Josephson showed that tunnelling of cooper pairs as similar as tunnelling of unpaired electrons.

**DC josephson effect**

The layer of insulator acts as a potential barrier for the Cooper pairs. A fraction of the current approaching this barrier from the left or the right manages to tunnel through the barrier and continue on the other side, without any net loss of energy. Thus, the junction permits the flow of a current even if the potential difference across it is zero.

The phenomenon in which the junction permits the flow of current without any net loss of energy even if the potential difference across it is zero is called *DC Josephson effect.*

*The above effect is due to* quantum mechanical tunnelling

The tunneling current is

$$I = I_C \sin\phi$$

The *super current J* of superconductor pairs across the junction depends on the phase difference δ. (Difference in phase between amplitude of electron pairs from one side of a junction to the other side)

(OR) $\varphi$ is phase difference between wave functions describing Cooper pairs.

The critical current depends on thickness and width of the layer.

**AC Josephson effect**

Let a voltage $V$ be applied across the Josephson junction. An electron pair experiences a potential energy difference $qV$ on passing across the junction, where $q = (-2e)$.

A pair electron on one side is at potential energy $-eV$ and the pair on the other side is at $eV$.

The energies of cooper pair on both side of the junction differ by $2eV$.

If we apply a $DC$ voltage $V$ across the junction, then it generates oscillate current (is $AC -$ current) across the junction. This is known as Ac Josephson effect

The frequency of this oscillating current is directly proportional to the voltage

⇒ Frequency of A.C. current or emitted EM radiations,

$\hbar\omega = 2eV \;\Rightarrow\; \omega = \frac{2eV}{\hbar}$ or $\omega = \frac{4\pi eV}{h}$ (OR)

In terms linear frequency

$h\nu = 2eV$ (or) $\nu = \frac{2eV}{h}$

A voltage of $10^{-6}\,V$ generates an AC of a frequency of 483.6 $MHz$

Switching time $\tau = \frac{1}{\nu} = \frac{h}{2eV}$

AC current $I = I_0 \sin(\omega t + \Phi)$

$$I = I_0 \sin\left(\frac{2eV}{\hbar}t + \Phi\right)$$

Thus, the measurement of the frequencies of AC Josephson currents can be used as a very precise and convenient method for the measurement of voltage.

Potentiometers based on this method attain a precision of about *1* part in $10^8$. This effect has also been utilized for the precise determination of the $\frac{\hbar}{e}$ value

At this state the junction emit or absorb radiation when the electron pair Cross the Junction.




## APPLICATIONS OF SUPERCONDUCTORS

### SQUID (Superconducting Quantum Interference Device)

SQUID stands for "Superconducting Quantum Interference Device"
Its one of the application of Josephson effect and the flux quantization.

SQUID is a device which can measure feeble magnetic field of the order of $10^{-15}\,T$, such as produced by the neural activity in human brain.

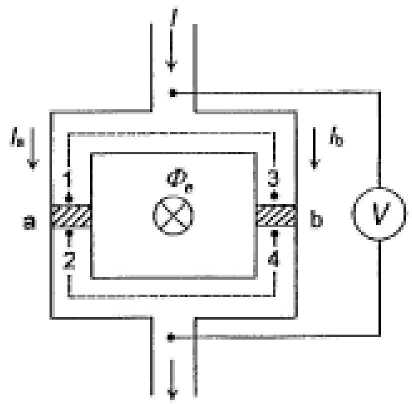
Principle: A DC magnetic field applied through a superconducting ring containing two Josephson junctions causes the maximum super current to show interference effects as a function of magnetic field intensity. This effect is used in SQUID.

i) SQUID is a sensitive magnetometer used to detect and measure weak magnetic fields. This forms an important application of Josephson junctions.

Description and working. SQUID is a double junction quantum interferometer. Two Josephson junctions mounted on a superconducting ring form this interferometer

The SQUID is based on the flux quantization in a superconducting ring. The total magnetic flux passing through the ring is quantized.

i) A small change in magnetic field produces variation in the quantum flux. When the magnetic field is applied perpendicular to the plane of the ring, current is induced at the two Josephson junctions. The induced current produces the interference pattern and it flows around the ring. The total magnetic flux passing through the ring is quantized.




Applications of SQUID

1) very minute magnetic signals are detected by the SQUID sensor.

2) SQUID is used as storage device for magnetic field.

3. The SQUID is used study tiny magnetic signals from the brain and heart.

4. SQUID magnetometer can detect the paramagnetic response in the liver and given the amount of iron held in the liver of the body accurately.

**2. Cryotron**

I) Cryotron is a magnetically operated current switch.

Principle. The superconducting property of a material disappears when the applied magnetic field is greater than the critical magnetic field ($H_c$).

A superconductor possesses two states, the superconducting and normal. The application of a magnetic field greater than $H_c$ can initiate a change from superconducting to normal and removal of the field reverses the process. This principle is applied in development of switching element cryotron.

**3. Magnetic levitation**

Principle. The diamagnetic property of a superconductor is the basis of magnetic levitation. When a material is superconducting, the magnetic flux lines will be expelled from the material (Meissner effect). So superconducting materials strongly repel external magnets.

If a small light and powerful magnet is kept over a superconducting material, the magnet will be levitated (lifted up) and it will float in air. This is called magnetic levitation.

Demonstration. Magnetic levitation can be demonstrated using a high $T_c$ superconductor like $Y_1Ba_2Cu_3O_7$ compound is cooled by pouring liquid nitrogen over it. The magnetic kept over it just floats in air.

Use of magnetic levitation

Magnetic levitation effect can be used for high speed transportation such as super fast trains, without frictional loss.

**Important Points**

**Characteristic of super conductor**

- Electrical resistivity $(\rho) = 0$
- Perfect diamagnetic $B_{in} = 0$
- Susceptibility $\chi = 0$
- $J = \sigma E \implies E = \frac{J}{\sigma} = 0$ i.e $E = 0$

Materials with the product of number of valence electron and the resistivity greater than $10^6$ $(n\rho > 10^6)$ show super connectivity.

Monovalent metals are generally not superconductors.

Good electrical conductors are not superconductors.

Super conductors are not good electrical conductor at room temperature

Amorphous thin films of Be, Bi and Fe shows superconductivity

Bi, Sb and Te are super conductor at high pressure

Metals with valance electron lies between 2 and 8 are normally super conductor

⇒ Cooper pair binding energy $10^{-8}eV$

⇒ Cooper pair energy gap $10^{-4}eV$

⇒ Stabilization energy in pure super conducting state is $\frac{Hc^2}{8\pi}$

⇒ Cooper pair act as a boson

The compound $ErRh_4B_4$ exhibits coexistence of superconducting and ferromagnetism.

⇒ Soft superconductors is a substance which exhibits reversible ideal magnetization and low $T_c$ whether Type 1 and type 2

⇒ Hard superconductors is a substance characterized by Irreversible magnetisation and high value of $T_c$ whether Type 1 or type 2.

⇒ In superconductor flux quantization is $\left(\frac{h}{2e}\right) = 2.75 \times 10^{-15} wb$

In superconducting state an energy gap $\Delta g = 3.5 k_B T_C$ separates conducting electrons from normal electrons

For type-1 superconductors the surface energy is always positive.

BCS theory assumed a spherical Fermi surface and isotropic energy gap

BCS theory valid for weak coupling superconductors

Properties does not change in superconducting transition

⇒ Reflection coefficient
⇒ Diffraction
⇒ Photoelectric property
⇒ Elastic property
⇒ volume property

} does not change

**Structure of Ferrites**

Ferrites spin orientations are similar to those for antiferromagnets but the magnetic moments of spin-up atoms and spin-down atoms are not equal. So the net magnetic moment is non-zero. Such materials are called ferrites. Which exhibit spontaneous magnetization below Neel temperature.

The resistivity of ferrites is very large; $10^{-2} - 10^{5} \Omega$ cm, which is $10^4 - 10^{10}$ times that of iron. This property controls the eddy currents so that not much heat is generated when such materials are placed in a frequently changing magnetic field. That is why ferrites find applications in telecommunication systems or in high frequency transformers. Due to their high resistivity and oxide nature; the ferrites are also termed as ceramic magnets.

The ferrites are iron oxides with the unit cell structure $(M^{2+}Fe_2^{3+}O_4)_8$ Where M indicates a divalent metal ion like $Fe^{2+}, Ni^{2+}, Co^{2+}, Mo^{2+}, Mg^{2+}, Cu^{2+}$, $Zn^{2+}$These atoms are arranged in a typical crystal structure called spinel. This structure is similar to that of $MgAl_2O_4$ commonly known as spinel compound. Hence Ferrites are said to have a spinel structure.